EXERCISE – II MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

1. If a line has a vector equation $\vec{r} = 2\hat{i} + 6\hat{j} + \lambda(\hat{i} - 3\hat{j})$, then which of the following statements hold goods ?

(A) the line is parallel to $2\hat{i} + 6\hat{j}$

(B) the line passes through the point $2\hat{i} + 3\hat{j}$

(C) the line passes through the point $\hat{i} + 9\hat{j}$

(D) the line is parallel to XY-plane

2. The vector $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$ is

(A) a unit vector

(B) makes an angle $\frac{\pi}{3}$ with the vector $2\hat{i} - 4\hat{j} - 3\hat{k}$

(C) parallel to the vector $-\hat{i} + \hat{j} - \frac{1}{2}\hat{k}$

(D) Perpendicular to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

3. The vector $\vec{c}$, directed along the external bisector of the angle between the vectors $\vec{a} = 7\hat{i} - 4\hat{j} + 4\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$ with $|\vec{c}| = 5\sqrt{6}$, is

(A) $\frac{5}{3}(\hat{i} - 7\hat{j} + 2\hat{k})$ (B) $\frac{5}{3}(\hat{i} + 7\hat{j} - 2\hat{k})$

(C) $\frac{5}{3}(-\hat{i} + 7\hat{j} - 2\hat{k})$ (D) $\frac{5}{3}(-\hat{i} + 7\hat{j} + 2\hat{k})$

4. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then the vectors $\vec{a} - \vec{b}$ and $\vec{b} - \vec{c}$ are

(A) collinear (B) linearly independent

(C) perpendicular (D) parallel

5. $\hat{a}$ and $\hat{b}$ are two given unit vectors at right angle. The unit vector equally inclined with $\hat{a}$, $\hat{b}$ and $\hat{a} \times \hat{b}$ will be

(A) $-\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} + \hat{a} \times \hat{b})$ (B) $\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} + \hat{a} \times \hat{b})$

(C) $\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} - \hat{a} \times \hat{b})$ (D) $-\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} - \hat{a} \times \hat{b})$

6. If a, b, c are different real numbers and $a\hat{i} + b\hat{j} + c\hat{k}, b\hat{i} + c\hat{j} + a\hat{k}$ and $c\hat{i} + a\hat{j} + b\hat{k}$ are position vectors of three non-collinear points A, B, and C, then

(A) centroid of triangle ABC is $\frac{a+b+c}{3}(\hat{i} + \hat{j} + \hat{k})$

(B) $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the three vectors

(C) perpendicular from the origin to the plane of triangle ABC meet at centroid

(D) triangle ABC is an equilateral triangle.

7. If $\vec{z}_1 = a\hat{i} + b\hat{j}$ and $\vec{z}_2 = c\hat{i} + d\hat{j}$ are two vectors in $\hat{i}$ and $\hat{j}$ system, where $|\vec{z}_1| = |\vec{z}_2| = r$ and $\vec{z}_1 . \vec{z}_2 = 0$, then $\vec{w}_1 = a\hat{i} + c\hat{j}$ and $\vec{w}_2 = b\hat{i} + d\hat{j}$ satisfy

(A) $|\vec{w}_1| = r$ (B) $|\vec{w}_2| = r$

(C) $\vec{w}_1 . \vec{w}_2 = 0$ (D) none of these

8. A line passes through a point A with position vector $3\hat{i} + \hat{j} - \hat{k}$ and parallel to the vector $2\hat{i} - \hat{j} + 2\hat{k}$. If P is a point on this line such that AP = 15 units, then the position vector of the point P is/are

(A) $13\hat{i} + 4\hat{j} - 9\hat{k}$ (B) $13\hat{i} - 4\hat{j} + 9\hat{k}$

(C) $7\hat{i} - 6\hat{j} + 11\hat{k}$ (D) $-7\hat{i} + 6\hat{j} - 11\hat{k}$

9. If $\vec{a}, \vec{b}$ are two non-collinear unit vectors and $\vec{a}, \vec{b}, x\vec{a} - y\vec{b}$ form a triangle, then

(A) $x = -1$; $y = 1$ and $|\vec{a} + \vec{b}| = 2\cos\left(\frac{\vec{a} \wedge \vec{b}}{2}\right)$

(B) $x=-1$; $y=1$ and

$\cos(\vec{a} \wedge \vec{b}) + |\vec{a} + \vec{b}| \cos(\vec{a} \wedge -(\vec{a} + \vec{b})) = -1$

(C) $|\vec{a} + \vec{b}| = -2\cot\left(\frac{\vec{a} \wedge \vec{b}}{2}\right)\cos\left(\frac{\vec{a} \wedge \vec{b}}{2}\right)$ and $x = -1$, $y=1$

(D) none of these

10. The volume of a right triangular prism $ABCA_1B_1C_1$ is equal to 3. If the position vectors of the vertices of the base ABC are A(1, 0, 1), B(2, 0, 0) and C(0, 1, 0), then position vectors of the vertex A_1 can be
(A) (2, 2, 2) (B) (0, 2, 0)
(C) (0, −2, 2) (D) (0, −2, 0)

11. Which of the following statement(s) is/are true ?
(A) If $\vec{n}.\vec{a} = 0, \vec{n}.\vec{b} = 0$ and $\vec{n}.\vec{c} = 0$ for some non-zero $\vec{n}$ then $[\vec{a}\vec{b}\vec{c}] = 0$
(B) there exist a vector having direction angles α = 30º and β = 45º
(C) locus of point for which x = 3 and y = 4 is a line parallel to the Z-axis whose distance from the Z-axis is 5
(D) the vertices of a regular tetrahedron are OABC where 'O' is the origin. Then vector $\overline{OA} + \overline{OB} + \overline{OC}$ is perpendicular to the plane ABC

12. If $\vec{r} = \hat{i} + 5\hat{j} + 5\hat{k} + \lambda(2\hat{i} + \hat{j} + 4\hat{k})$ and $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) = 3$ are the equations of a line and plane respectively, then which of the following is false ?
(A) line is perpendicular to the plane
(B) line lies in the plane
(C) line is parallel to the plane but does not lie in the plane
(D) line cuts the plane is one point only

13. Unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar. A unit vector $\vec{d}$ is perpendicular to them. If $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \frac{1}{6}\hat{i} - \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k}$, and the angle between $\vec{a}$ and $\vec{b}$ is 30º, then $\vec{c}$ is
(A) $(\hat{i} - 2\hat{j} + 2\hat{k})/3$ (B) $(\hat{i} - 2\hat{j} + 2\hat{k})/3$
(C) $(-2\hat{i} - 2\hat{j} - \hat{k})/3$ (D) $(-\hat{i} + 2\hat{j} - 2\hat{k})/3$

14. Let $\vec{p} = 2\hat{i} + 3\hat{j} - a\hat{k}$, $\vec{q} = b\hat{i} + 5\hat{j} - \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 3\hat{k}$. If $\vec{p}, \vec{q}, \vec{r}$ are coplanar and $\vec{p}.\vec{q} = 20$, then a and b have the values
(A) 1, 3 (B) 9, 7 (C) 5, 5 (D) 13, 9

15. The value(s) of $\alpha \in [0, 2\pi]$ for which vector $\vec{a} = \hat{i} + 3\hat{j} + (\sin 2\alpha)\hat{k}$ makes an obtuse angle with the z-axis and the vectors $\vec{b} = (\tan\alpha)\hat{i} - \hat{j} + 2\sqrt{\sin\frac{\alpha}{2}}\hat{k}$ and $\vec{c} = (\tan\alpha)\hat{i} + (\tan\alpha)\hat{j} + 3\sqrt{\operatorname{cosec}\frac{\alpha}{2}}\hat{k}$ are orthogonal, is/are
(A) $\tan^{-1} 3$ (B) $\pi - \tan^{-1} 2$
(C) $\pi + \tan^{-1} 3$ (D) $2\pi - \tan^{-1} 2$

16. If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{b} = y\hat{i} + z\hat{j} + x\hat{k}$ and $\vec{c} = z\hat{i} + x\hat{j} + y\hat{k}$, then $\vec{a} \times (\vec{b} \times \vec{c})$ is
(A) parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$
(B) orthogonal to $\hat{i} + \hat{j} + \hat{k}$
(C) orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$
(D) orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

17. If $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors satisfying the condition $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$, then
(A) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal is pairs (B) $[\vec{a}, \vec{b}, \vec{c}] = [\vec{a}]^2$
(C) $[\vec{a}\,\vec{b}\,\vec{c}] = |\vec{c}|^2$ (D) $|\vec{b}| = |\vec{c}|$